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# $B_c$ decays from highly improved staggered quarks and NRQCD

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We calculate semileptonic form factors for the decays  $B_c \rightarrow \eta_c l \nu$  and  $B_c \rightarrow J/\psi l \nu$  over the entire  $q^2$  range, using a highly improved lattice quark action for charm at several lattice spacings down to  $a = 0.045$  fm. We have two ways of treating the  $b$  quark: either with an  $\mathcal{O}(\alpha_s)$  improved NRQCD formalism or by extrapolating a heavy mass  $m_h$  to  $m_b$  in the relativistic formalism. Comparison of the two approaches provides an important cross-check of methodologies in lattice QCD. Nonperturbative renormalisation of the currents in the relativistic theory also allows us then to fix NRQCD-charm normalisation for  $b$  to  $c$  decays such as  $B \rightarrow D$  and  $B \rightarrow D^*$ .

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## 1. Introduction

Many properties of the  $B_c$  meson will be measured for the first time at the LHC. Indeed the LHCb collaboration has already measured the  $B_c$  lifetime in the semileptonic channel  $B_c \rightarrow J/\psi \mu \nu$  [1], with measurements of  $B_c \rightarrow J/\psi \tau \nu$  underway. Therefore it is desirable to have lattice QCD calculations of the form factors parameterising these processes. A reliable calculation of the ratio  $R(B_c \rightarrow J/\psi)$  along with experimental measurements may shed light on existing anomalies in  $R(B \rightarrow D)$  and  $R(B \rightarrow D^*)$  [2, 3, 4, 5].

Simulating heavy quarks is a challenge for lattice calculations due to discretisation artifacts which grow as  $(am_h)^n$ , where  $a$  is the lattice spacing and  $m_h$  is the mass of a heavy quark. By using a highly improved staggered quark (HISQ) action we are able to simulate charm quarks with controlled discretisation effects and on the finest available lattices ( $a \approx 0.045$  fm) simulate near to the bottom mass. We can also calculate the form factors with a complementary approach working directly at  $m_b$  using an improved non-relativistic (NRQCD) effective theory formalism. This framework is also being used in other calculations with a  $b \rightarrow c$  transition in particular  $B \rightarrow D^*$  and  $B \rightarrow D$ , and others. A useful output of this calculation will be to assess the systematic uncertainties present in that framework.

## 2. Methodology

We study form factors for the decays  $B_c \rightarrow \eta_c l \nu$  and  $B_c \rightarrow J/\psi l \nu$ . These can be determined from matrix elements of the  $V - A$  operator between the states of interest. The matrix elements are expressed in terms of the form factors as

$$\langle \eta_c(p) | V^\mu | B_c(P) \rangle = f_+(q^2) \left[ P^\mu + p^\mu - \frac{M^2 - m^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M^2 - m^2}{q^2} q^\mu \quad (2.1)$$

and

$$\begin{aligned} \langle J/\psi(p, \epsilon) | V^\mu - A^\mu | B_c(P) \rangle = & \frac{2i\epsilon^{\mu\nu\rho\sigma}}{M+m} \epsilon_\nu^* p_\rho P_\sigma V(q^2) - (M+m) \epsilon^{*\mu} A_1(q^2) + \\ & \frac{\epsilon^* \cdot q}{M+m} (p+P)^\mu A_2(q^2) + 2m \frac{\epsilon^* \cdot q}{q^2} q^\mu A_3(q^2) - 2m \frac{\epsilon^* \cdot q}{q^2} q^\mu A_0(q^2). \end{aligned} \quad (2.2)$$

Here  $q = P - p$  is the difference in four-momentum between the  $B_c$  and the outgoing hadron. We work in the frame where the  $B_c$  is at rest.  $q_{\max}^2$  corresponds to the outgoing hadron being produced at rest and  $q^2 = 0$  corresponds to maximum recoil of the outgoing hadron. The form factors are calculated using two different formalisms, which are described in more detail below.

In the fully relativistic calculation, the vector current normalisation is determined using the relation

$$\langle \eta_c(p) | S | B_c(P) \rangle = \frac{M^2 - m^2}{m_{b0} - m_{c0}} f_0(q^2),$$

which is absolutely normalised. Comparing this to Eq. (2.1) at  $q_{\max}^2$ , where only the  $f_0$  term contributes, determines the normalisation of  $V^\mu$ . We have normalised the axial vector current using the PCAC relation

$$p_\mu \langle 0 | A^\mu | B_c \rangle = (m_{c0} + m_{b0}) \langle 0 | P | B_c \rangle,$$

and the form factor  $A_1(q^2)$  is extracted from the three-point matrix element (2.2) by arranging that  $\varepsilon^* \cdot q = 0$ .

We use the highly improved staggered quark (HISQ) action [6] which systematically removes lattice artifacts and in particular allows for the simulation of charm quarks with small discretisation effects [7]. On the finer sets of ensembles we can simulate at masses well beyond  $m_c$  [8]. By working in a regime of  $am_h \lesssim 0.8$  on successively finer lattices, we calculate the physics of interest over a range in  $m_h$  and extrapolate the results to  $m_b$ .

Alternatively we can make use of an improved non-relativistic effective theory formalism (NRQCD) [9] for which we can work directly at the  $b$  mass. In this case the current operators mediating the decay have a relativistic expansion, e.g. for the spatial axial-vector current

$$A_k^{\text{nrqcd}} = (1 + \alpha_s z_k^{(0)} + \dots) \left[ A_k^{(0)} + (1 + \alpha_s z_k^{(1)}) A_k^{(1)} + \alpha_s z_k^{(2)} A_k^{(2)} + \dots \right],$$

where  $A_k^{(1)}$ ,  $A_k^{(2)}$ , ... are higher order current corrections, with matrix elements proportional to  $1/m_b$ . An analogous expression holds for the vector current. The matching coefficient  $z_k^{(0)}$  has been computed in QCD perturbation theory [10, 11], and there is a systematic uncertainty from unknown  $\mathcal{O}(\alpha_s^2)$  terms. This is a leading uncertainty in the ongoing calculation of  $B \rightarrow D^*$  [12], which feature the same  $b \rightarrow c$  currents. A comparison of results from the relativistic extrapolation method, where the normalisation is simpler and does not rely on perturbation theory, will give an important handle on this uncertainty.

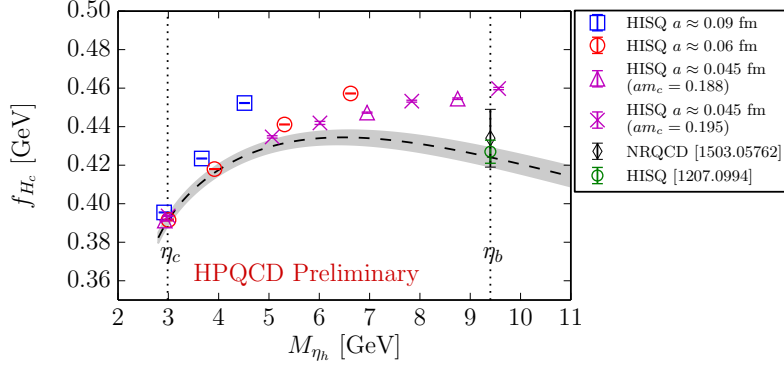
Calculations are performed on  $n_f = 2 + 1 + 1$  HISQ gauge configurations generated by the MILC collaboration [13], with lattice spacings  $a \approx 0.15, 0.12, 0.09, 0.06$ , and  $0.045$  fm. All results presented here are on ensembles with  $m_s/m_{ud} = 5$ , i.e. unphysically heavy pions. Since the processes studied only have valence heavy quarks, light quark mass dependence is expected to be small.

### 3. Results

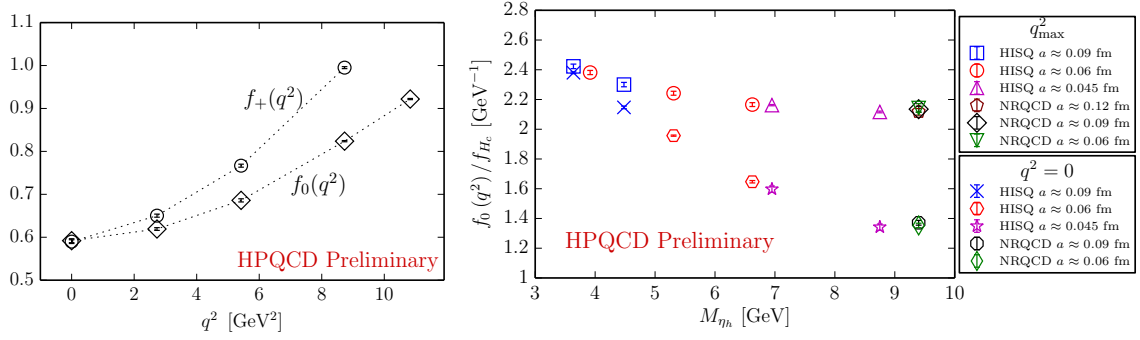
Fig. 1 shows results for the decay constant of a pseudoscalar meson composed of one charm quark and a heavy quark for  $m_c < m_h < m_b$  at various lattice spacings. For this quantity it is clear that discretisation effects become sizable as  $m_h$  is increased. Nevertheless these effects remain under control all the way to the  $b$  mass on the finest ensemble with  $a \approx 0.045$  fm, and the continuum extrapolation of the combined data is shown as a grey band in the figure.

Fig. 2 and Fig. 3 show results for the  $B_c \rightarrow \eta_c$  form factors  $f_+(q^2)$  and  $f_0(q^2)$  using both the NRQCD and relativistic formalisms. Fig. 2 (left) shows the result for both form factors from NRQCD calculated on the  $a \approx 0.09$  fm ensembler, over the full  $q^2$  range. Fig. 2 (right) shows the extrapolations in  $m_h$  for the points  $f_0(q_{\text{max}}^2)/f_{H_c}$  and  $f_0(q^2 = 0)/f_{H_c}$  from the relativistic data compared to the NRQCD results from multiple lattice spacings. The  $f_0(q_{\text{max}}^2)/f_{H_c}$  extrapolation is shown in more detail in Fig. 3 and includes the continuum fit to the relativistic data. It is clear from the figure that discretisation effects are small for this quantity throughout the ranges studied, and that the continuum result is compatible with the result at  $m_b$  coming from NRQCD.

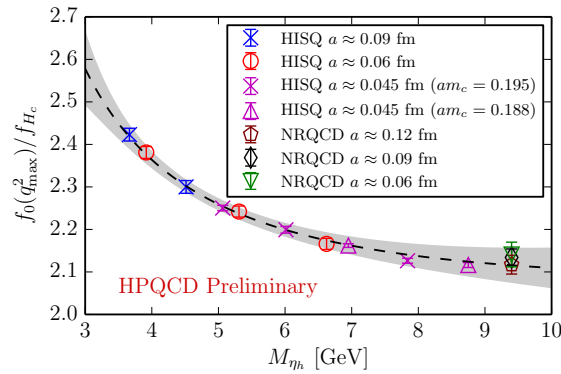
Fig. 4 shows NRQCD results for the  $B_c \rightarrow J/\psi$  form factors  $A_1(q^2)$  and  $V(q^2)$  on the  $a \approx 0.09$  fm ensemble. Extrapolations of the relativistic data in  $m_h$  are shown in Fig. 5 for the points



**Figure 1:**  $f_{H_c}$  vs.  $M_{\eta_h}$  computed on fine, superfine, and ultrafine ensembles. For each ensemble  $m_h$  is varied from  $m_c$  to  $am_h = 0.8$ . The continuum fit result is shown as a grey band. The results for  $f_{B_c}$  are from NRQCD [11] and HISQ on  $n_f = 2 + 1$  ensembles [8].

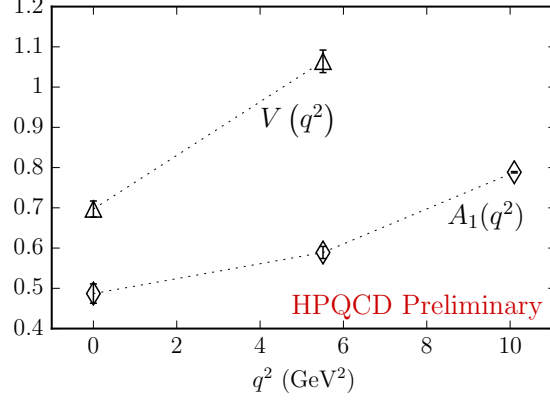


**Figure 2:** (Left)  $B_c \rightarrow \eta_c$  form factors  $f_0(q^2)$  and  $f_+(q^2)$  calculated using lattice NRQCD, determined over the full  $q^2$  range. (Right) Results for  $f_0(q^2_{\max})/f_{H_c}$  and  $f_0(0)/f_{H_c}$  using the HISQ formalism as  $m_h$  is increased towards  $m_b$ . The rightmost points are the corresponding NRQCD results at the physical  $b$  mass.

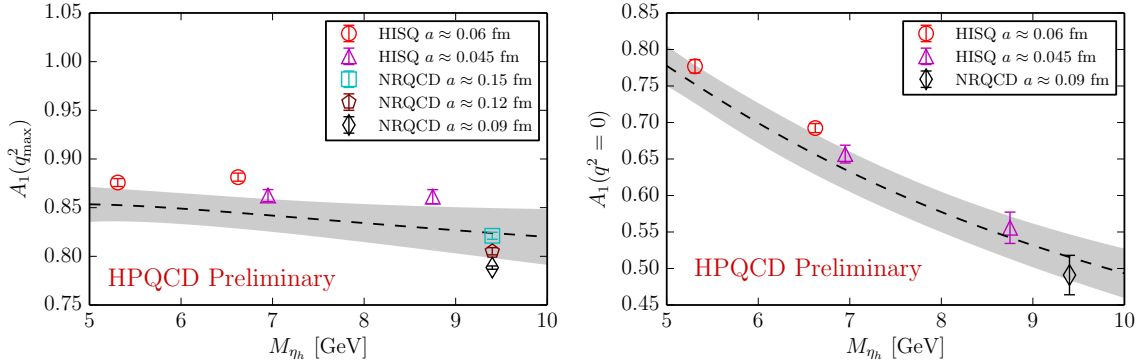


**Figure 3:**  $f_0(q^2_{\max})/f_{H_c}$  vs.  $M_{\eta_h}$  in the HISQ formalism on fine, superfine, and ultrafine ensembles. The rightmost points are NRQCD determinations at the physical  $b$  mass. The grey band shows the HISQ results extrapolated to the continuum.

$A_1(q_{\text{max}}^2)$  (left) and  $A_1(q^2 = 0)$  (right), along with the NRQCD results at  $m_b$  from multiple lattice spacings. As in the case of the  $B_c \rightarrow \eta_c$  form factors, there is good agreement between NRQCD results and the continuum extrapolations of the relativistic data.



**Figure 4:**  $B_c \rightarrow J/\psi$  form factors  $A_1(q^2)$  and  $V(q^2)$  calculated on the  $a \approx 0.09$  fm ensemble using lattice NRQCD.



**Figure 5:** (Left) Extrapolation in heavy quark mass  $m_h$  for the  $B_c \rightarrow J/\psi$  form factor  $A_1(q_{\text{max}}^2)$  using the relativistic HISQ formalism. The rightmost points are NRQCD results at the physical  $b$  mass, and the grey band shows the continuum extrapolation of the HISQ results. (Right) HISQ results for  $A_1(q^2 = 0)$  vs.  $M_{\eta_h}$  along with the NRQCD result computed at the physical  $b$  mass. The grey band shows the continuum extrapolation of the HISQ results.

## 4. Conclusions

We are calculating the form factors for the  $B_c$  semileptonic decays  $B_c \rightarrow \eta_c l \nu$  and  $B_c \rightarrow J/\psi l \nu$ , using two complementary approaches. One approach utilises the HISQ action on successively finer lattices to simulate heavy quarks approaching the  $b$  mass. The other works directly at the  $b$  mass with improved NRQCD effective theory. In both approaches we are able to obtain a signal over the full  $q^2$  range, and we see a good agreement between the results of each method.

The NRQCD  $b \rightarrow c$  currents are also being used in computations of  $B \rightarrow D^*$  and  $B \rightarrow D$ . Understanding more precisely the normalisations of the NRQCD  $b \rightarrow c$  currents using nonperturbative information provided by the fully relativistic computation will improve the analysis of this data.

## 5. Acknowledgements

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